Journal of Engineering Physics and Thermophysics, Vol. 67, No. 1-2, 1994

THEORY OF THE CURRENT TO A PLANE WALL PROBE IN THE REGIME OF A CONTINUOUS MEDIUM

V. A. Kotel'nikov and A. V. Shan'kov

UDC 533.9

A method is suggested for calculating the total current to a plane wall probe located in the side surface of a hypersonic aircraft. The applicability of the method is limited by weaker conditions than those used in the well known works [1]. Comparison of the results of probe experiments with those obtained from SHF inspection of the plasma and with calculations of other authors in the region of their applicability demonstrated satisfactory agreement.

1. In the present work we suggest a method for calculating the total current to a plane wall probe located in a flow of a dense weakly ionized gas. Thus, the regime of a continuous medium is realized. The parameters of the boundary layer not perturbed by the probe are considered to be given. We also know the geometric dimensions and the potential of the probe (for definiteness, we will consider it negative).

The problem of finding the probe current may turn out to be one-dimensional (the characteristic dimension of the probe is many times greater than the thickness of the boundary layer and the space charge layer), two-dimensional (the geometric dimensions of the proble have been selected so that the probe current is constant along one of the coordinates), and three-dimensional. At the present time, a method has been developed for calculating a three-dimensional nonstationary probe probem [2], but in many practical cases it is sufficient to be confined to a two-dimensional statement. This saves substantial computer time in numerical experiments.

Among the engineering methods used to calculate the current to a plane wall probe, the formula of Chung [1] is generally useful, but it is applicable only in the case of a thin space charge layer. It permits one to obtain rather good accuracy in processing probe characteristics if the ratio of the boundary layer thickness to the Debye radius ratio exceeds 10.

We suggest another formula for calculating the ion current to a plane wall probe. It is valid for both a thin and a thick space charge layer.

2. We consider a plane wall probe with the characteristic dimension r_p and the potential φ_p that is located on a plate with a sharp leading edge or an axisymmetric blunt body immersed in a flow of a weakly ionized plasma with frozen chemical reactions. Suppose that the boundary layer thickness in the region of the location of the probe δ is of the same order of magnitude as the space charge layer thickness Δ.

The problem of finding the probe current as a function of its potential reduces to the solution of a three-dimensional nonstationary problem that includes continuity equations for the ions and the electrons, an energy equation for the electrons, and a Poisson equation for the self-consistent electric field. Let us write the system in dimensionless form [1–3]:

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \mathbf{I}_e = 0 , \quad \mathbf{I}_e = - D_e \left(\frac{1}{T_e} \frac{\partial n_e T_e}{\partial \mathbf{r}} + \varepsilon n_e \mathbf{E} \right) , \tag{1}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \mathbf{I}_i = 0 , \quad \mathbf{I}_i = - D_i \left(\frac{1}{T_i} \frac{\partial n_i T_i}{\partial \mathbf{r}} + n_i \mathbf{E} \right) , \tag{2}$$

Sergo Ordzhonikidze Moscow Aviation Institute, Moscow, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 67, Nos. 1-2, pp. 119-122, July-August, 1994. Original article submitted October 7, 1992.

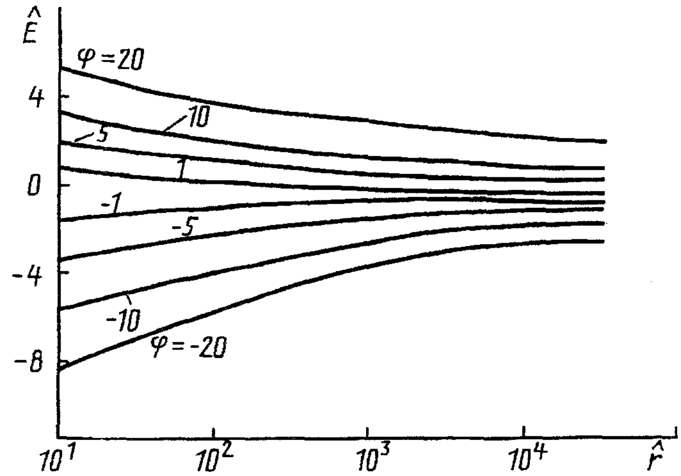


Fig. 1. Dependence of the electric field strength $\hat{E}$ on the probe potential $\hat{\varphi}$ and its size $\hat{r}$.

$$n_e \frac{\partial H_e}{\partial t} - n_e \varepsilon \frac{\partial \varphi}{\partial t} - \frac{\partial}{\partial t}(n_e T_e) + \mathbf{I}_e \frac{\partial H_e}{\partial \mathbf{r}} = \frac{\partial}{\partial \mathbf{r}}\left(\Lambda_e \frac{\partial T_e}{\partial \mathbf{r}}\right), \tag{3}$$

$$\frac{\partial}{\partial \mathbf{r}} \mathbf{E} = n_i - n_e, \quad \mathbf{E} = -\frac{\partial \varphi}{\partial \mathbf{r}}. \tag{4}$$

In system (1)-(4), $n_{e,i}$, $T_{e,i}$ are the concentrations and temperatures of the ions and electrons; H_e is the enthalpy of the electrons; E, φ are the strength and potential of the self-consistent electric field. The remaining symbols are generally accepted and correspond to [1–3].

The system of equations (1)-(4) was supplemented with boundary and initial conditions [1–3] and solved by the method of successive iterations in time using the algorithm of large particles for the continuity equations, the schemes of the arithmetic mean for the energy equation, and the Pisman–Rachford for the Poisson equation [1–3]. The profiles of the concentrations, temperatures, and velocities of the neutral component in the boundary layer were assumed to be given.

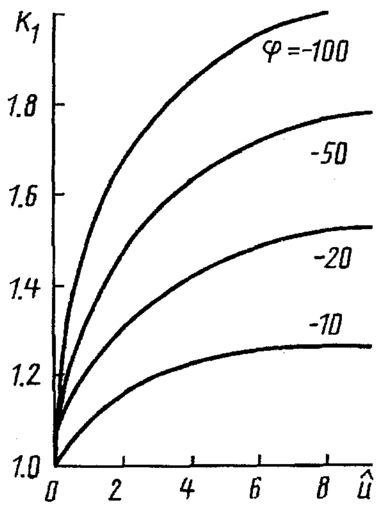
On the basis of numerical experiments we obtained the dependence of the probe current on the plasma free stream velocity, the characteristic dimension of the probe, its potential, and other parameters of the problem. This made it possible to obtain a simple computational formula for processing the characteristics of the plane wall probe. If we denote the ratio between the density of the current to a probe in the presence of a directed velocity and the density of the current to the same probe in a quiescent plasma by K_1 and the ratio between the density of the current to a probe with the characteristic dimension r_p and the density of the current to a probe of very large size by K_2, then by dividing the experimentally measured density of the probe current by K_1K_2, we obtain the density of the current to a probe operating under the idealized conditions that the direction of the velocity is immaterial and the probe is of infinitely large size. Under these conditions the density of the probe current depends mainly on the potential gradient and the concentration of the ions [1, 3]:

$$\frac{I_{exp}}{K_1 K_2} = I_{i0} = eD_i \left(\frac{\partial n_i}{\partial y} + \frac{e n_i}{kT_i} E \right), \tag{5}$$

where y is the coordinate across the boundary layer.

Equation (5) yields a formula for the concentration of the ions on the outer edge of the space charge layer:

$$n_{i\Delta} = \left[\frac{I_{i0}/eD_i - \partial \hat{n}_i / \partial \hat{y}}{e\hat{E}\,(4\pi/kT_i)^{1/2}} \right]^{2/3}, \tag{6}$$



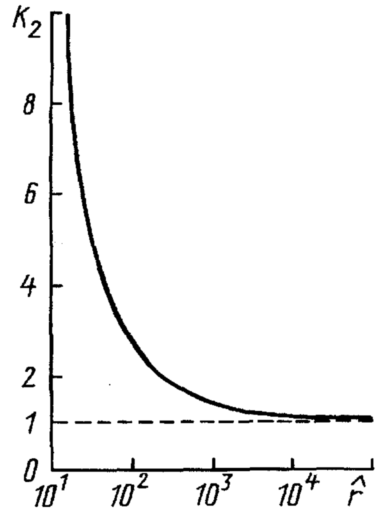


Fig. 2. Dependence of the coefficient K_1 on the free stream velocity $\hat{u}$.

Fig. 3. Dependence of the coefficient K_2 on the probe size $\hat{r}$.

where the sign Λ denotes dimensionless quantities:

$$\hat{E} = \frac{E}{M_E} = \frac{E}{kT_i/er_D}; \quad \hat{n}_i = \frac{n_i}{n_{i\Delta}}; \quad \hat{y}_i = \frac{y_i}{r_D}; \quad r_D = \left(\frac{kT_i}{4\pi ne^2}\right)^{1/2}.$$

The values of $\hat{E}$, K_1, and K_2 were found in a numerical experiment; they are presented in Figs. 1–3.

To carry out calculations by formula (6), the derivative $\partial\hat{n}_i/\partial\hat{y}$ should be determined from additional considerations. If the concentration profile in a boundary layer changes smoothly, then the following condition is usually fulfilled:

$$\frac{en_i E}{kT_i} >> \frac{\partial n_i}{\partial y}, \tag{7}$$

and therefore in approximate calculations the concentration gradient can be neglected.

3. Formula (6) was checked by comparing results of probe experiments with results of experiments on SHF inspection of a plasma as well as with Chung's calculations [1] in the region of its applicability. The comparisons showed satisfactory coincidence, which makes it possible to recommend formula (6) for practical application.

NOTATION

$n_{i,e}$, concentration of ions and electrons; $T_{i,e}$, temperature of ions and electrons; H_e, enthalpy of electrons; E, electric field strength; φ, electric field potential; $I_{i,e}$, density of ion and electron current; $D_{i,e}$, coefficient of diffusion for ions and electrons.

REFERENCES

1. P. M. Chung, L. Talbot, and K. J. Touryan, Electric Probes in Stationary and Flowing Plasmas: Theory and Application, Springer-Verlag, Berlin, Heidelberg, New York (1975).
2. V. P. Demkov, Mathematical Modeling of Transfer Processes in a Plasma with Account for Surface Processes, Candidate's Dissertation (Phys. and Math.), Moscow (1989).
3. B. V. Alekseev and V. A. Kotel'nikov, The Probe Method of the Diagnostics of a Plasma [in Russian], Moscow (1988).